MATH 374-001 Test 2

March 5, 2014

Name: _____

Directions: You have 50 minutes to complete the following exam. Show all applicable work. Answers without proper evidence of understanding will not receive credit.

1. (5 pts.) Prove or disprove the following statement.

If x and y are rational numbers, then the sum $x + y$ is a rational number.

2. (5 pts.) Prove or disprove the following statement.

For every natural number n, the number $2^n + 1$ is prime.

3. (5 pts.) Prove or disprove the following statement.

For every integer n, if n^2 is an odd number, then n is an odd number.

4. (5 pts.) Write a recursive definition for the set S of all binary strings (strings of 0's and 1's) which do not contain the string 00. For example, the strings λ (the empty string), 0, 1, 01, 10, 11, 010, 011, 101, 110, 111, 0101, 0110, 1010, 0111, 1011, 1101, 1110, 1111 are all members of S, while 00, 001, 100 are not.

5. (5 pts.) The following program segment computes $n!$. Show that the program is correct by

(a) proving that $Q : \{y = (x-1)!\}$ is a loop invariant and

(b) evaluating the postcondition at loop termination.

$FACT(n) :$

$\quad x = 2$

$\quad y = 1$

$\quad$ **while** $x \neq n+1$ **do**

$\qquad y = y * x$

$\qquad x = x+1$

$\quad$ **end while**

$\quad$ return y

6. (6 pts.) For the following recurrence relation, write the first 5 terms of the sequence, find a closed formula for $T(n)$, and prove the formula is true.

$$T(1) = 1$$
$$T(n) = 2T(n-1) + 1 \text{ for } n > 1.$$

Honor Statement:

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code. As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Signature: _____